Performance Analysis of IEEE 802.11.ad Downlink Hybrid Beamforming

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Abstract—Hybrid beamforming (BF) is a widely considered strategy to enable downlink multiuser transmission for mmWave communication systems. However, current mmWave WiFi standard, the IEEE 802.11 ad, does not support hybrid BF because it could only serve one user at a time. Thus, it is important to implement hybrid BF based on IEEE 802.11.ad such that it could be applied in future mmWave WiFi. In this paper, we propose a hybrid BF scheme compatible with the IEEE 802.11 ad, and then analyze its BF overhead and throughput gain. Theoretical analysis and simulation results show that with the increasing number of users, BF overhead increases linearly, whereas the throughput gain increases and then decreases. Specifically, we analyze the tradeoff between the hybrid BF overhead and the throughput gain of multiuser transmission enabled by hybrid BF based on an IEEE 802.11 ad setting. Our finding suggests that there is an optimal number of users that a hybrid BF enabled mmWave communication systems should support.

I. Introduction

Millimeter-wave (mmWave) band communication, particularly the 60 GHz band, has received considerable attention for short-range indoor applications, such as wireless personal area network (WPANs) and wireless local area networks (WLANs)[1][2][3]. For example, the IEEE 802.15.3c has been proposed for WPANs, and the IEEE 802.11.ad for WLANs, both of which are operated in 60GHz band. With a wide bandwidth, mmWave communications can achieve transmission data rate higher than gigabits per second. The high data rate makes mmWave WiFi systems promising to support applications such as wireless docking, high definition video transmission and real time gaming for multiple users simultaneously [4][5]. Multiuser downlink transmission is desired for higher system throughput. In mmWave communication systems, a large antenna array is utilized to obtain directional antenna gain to counter the extremely high pathloss. However, this poses challenges to support multiuser transmission via conventional digital multiuser beamforming (BF), which relied on channel state information (CSI) exchange between transmitter and receiver [6][7]. For mmWave systems with a large antenna array, conventional digital BF would incur high overhead and excessive power consumption with fully digital precoding [8]. Thus, an efficient multiuser BF scheme is desired for the mmWave communication systems.

Hybrid BF is considered as a low-complexity strategy to achieve multiuser BF in mmWave communication systems [9]. Hybrid BF consists of an analog BF and a digital BF. Different from digital BF, the analog BF controls the phase of the transmitted signal at each antenna via analog phase shifters. It has inferior performance for limited array gain, and is adopted in the IEEE 802.11.ad via sector level sweep phase. Hybrid BF, on the other hand, can achieve close to the optimal fully-digital BF with much lower complexity [10]. Thus, it would be of great benefit if hybrid BF could be applied to mmWave WiFi for multiuser downlink transmission. Previous works have proposed to apply hybrid BF in the mmWave systems. A hybrid precoding in wideband mmWave communication systems with a feedback between transmitter and receiver is proposed by A. Alkhateeb et al. in [11]. The sparsity of received signal is exploited to design hybrid precoding in [12]. An extended work in presented in the mmWave cellular network [13]. However, how to implement hybrid BF based on the IEEE 802.11.ad standard, and how much overhead would incur is not well studied, which hinges the adoption of hybrid BF in mmWave WiFi.

In this paper, we propose a hybrid BF protocol to support hybrid BF strategy based on the IEEE 802.11.ad, and analyze the overhead of BF as well as the throughput gain. Analysis demonstrates that the overhead of BF increases linearly with the number of users, which reduces the throughput gain when user number is large. With the increase of users, the throughput gain increases and then decreases. Thus, our finding suggests that there is an optimal number of users that a hybrid BF enabled mmWave systems should support.

The remainder of the paper is organized as follows. In Section II, we present the BF protocol in IEEE 802.11.ad. In Section III, system model of downlink multiuser transmission is given and hybrid BF algorithm is presented. In Section IV, the proposed hybrid BF protocol is given. Section V presents the analysis of overhead of our proposed hybrid BF protocol. In Section VI, simulations results are presented. In Section VII, concluding remarks are given.

II. BF protocol in IEEE 802.11.ad

In this section, we introduce the BF protocol in IEEE 802.11.ad. As the beacon interval structure shown in Fig. 1, BF protocol consists of sector level sweep (SLS), announcement transmission interval (ATI) and beam refinement protocol (BRP).
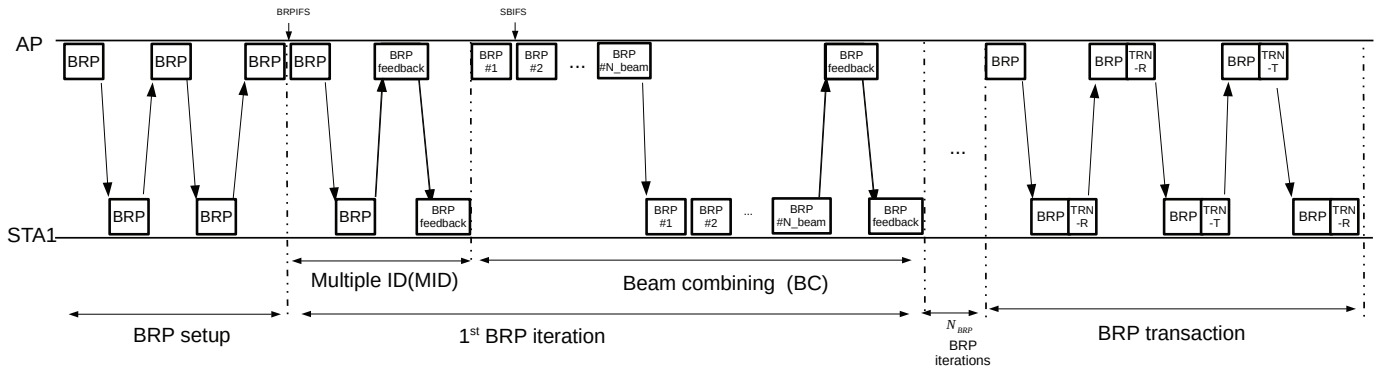
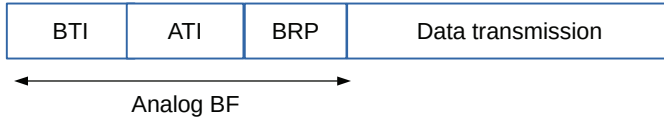Fig. 3. BRP in IEEE 802.11.ad.

Fig. 1. Beacon interval structure in IEEE 802.11.ad.

To compensate pathloss, directional antennas have been adopted at both transmitter and receiver. In SLS, both transmitter and receiver sweep all the sectors to obtain the optimal sector which is the coarse beam. Further, in BRP, transmitter and receiver communicate iteratively with BRP frames to obtain the refined beams. ATI is used for channel access information announcement.

As shown in Fig. 2, SLS consists of initial sector sweep (ISS), responder sector sweep (RSS), sector sweep feedback and sector sweep ACK. In ISS, access point (AP) transmits sector sweep (SSW) frames in different transmitting sectors and the station (STA) uses omni directional antennas to receive signal. By comparing signal strength at the STA, the optimal transmitting sector is known. The optimal transmitting sector ID is fed back to AP via sender sweep ACK. RSS follows the same operations to obtain the ID of the optimal receiving sector.
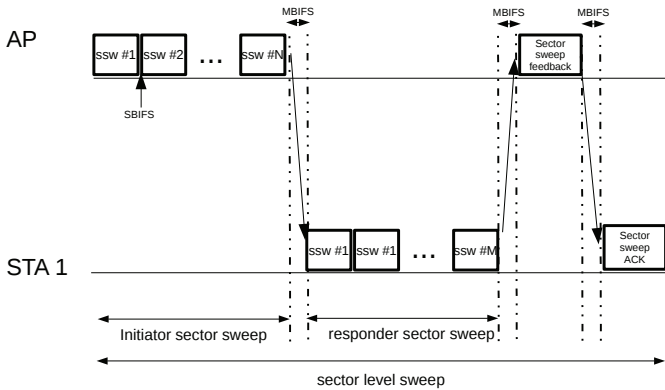

Fig. 2. SLS in IEEE 802.11.ad.

BRP is an iterative algorithm to obtain refined beams, which is made up of BRP setup, multiple sector ID (MID), beam combining (BC) and BRP transaction, as shown in Fig. 3. BRP setup allows STA to exchange beam refinement capability in-

formation and request the execution of other BRP sub-phases. Due to imperfect quasi-omni receive antenna patterns, MID and BC are optionally used to find better initial antenna weight vector (AWV) for the iterative beam refinement. We perform N_{BRP} iterations to finish MID and BC. BRP transaction is the final subphase which exchanges information between AP and STA.

III. HYBRID BF FOR DOWNLINK MULTIUSER TRANSMISSION

In this section, we present the system model and algorithm of hybrid BF for downlink multiuser transmission.

A. System Model of Hybrid BF

To implement downlink multiuser transmission, system should have the following structure [9]. Unlike IEEE 802.11.ad systems that both transmitter and receiver only have analog BF, we have a digital BF at AP which is used to mitigate multiuser interference.
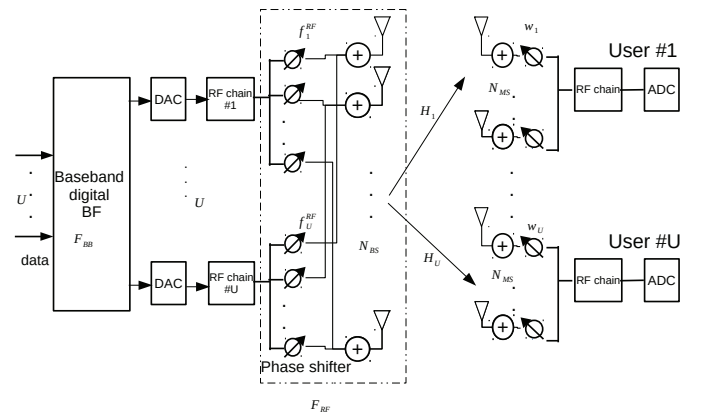

Fig. 4. Structure of hybrid BF.

Let N_s, N_{RF}, N_{BS} and N_{MS} denote the number of transmitted data streams, RF chains, antennas at AP and antennas at each user, respectively. For simplicity, we can consider users are equipped with the same number of antennas. To support U user, we should have at least U RF chains at AP and U data streams to be transmitted. For simplicity, let $N_s = N_{RF} = U$ [9].

Digital BF at AP can be written as $\mathbf{F}_{BB} = [\mathbf{f}_1^{BB}, \mathbf{f}_2^{BB}, ..., \mathbf{f}_U^{BB}] \in \mathbb{C}^{U \times U}$ and analog BF at AP is $\mathbf{F}_{RF} = [\mathbf{f}_1^{RF}, \mathbf{f}_2^{RF}, ..., \mathbf{f}_U^{RF}] \in \mathbb{C}^{N_{BS} \times U}$. The transmitted signal is

$$\mathbf{x} = \mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s},$$

where $\mathbf{s} = [s_1, s_2, ..., s_U]^T \in \mathbb{C}^{U \times 1}$ is the transmitted symbols for different users whose power is $\mathbb{E}[|s_i|^2] = \frac{P}{U}$. $\mathbf{A}^H$ and $\mathbf{A}^T$ stand for conjugate transpose and transpose of $\mathbf{A}$ respectively. $\mathbf{a}^*$ represents the conjugate of vector $\mathbf{a}$ in this paper.

Let $\mathbf{H_i} \in \mathbb{C}^{N_{MS} \times N_{BS}}$ represent the channel matrix of communication link between AP and the user i. For simplicity, we consider that there is only one path between AP and the user, $\mathbf{H_i}$ is given by [9]

$$\mathbf{H_i} = \sqrt{N_{MS}N_{BS}}a\alpha_i(\theta)\alpha_{BS}^*(\phi), \qquad (1)$$

where a is the complex gain of the path. $\alpha_i(\theta)$ and $\alpha_{BS}(\phi)$ are the antenna array response vectors of user i and AP. For simplicity, we adopt uniform linear arrays in all devices, thus $\alpha_i(\theta)$ and $\alpha_{BS}(\phi)$ can be represented as

$$\alpha_i(\theta) = \frac{1}{\sqrt{N_{MS}}}[1, e^{j\frac{2\pi}{\lambda}r\sin(\theta)}, ..., e^{j(N_{MS}-1)\frac{2\pi}{\lambda}r\sin(\theta)}]^T,$$

$$\alpha_{BS}(\phi) = \frac{1}{\sqrt{N_{BS}}}[1, e^{j\frac{2\pi}{\lambda}r\sin(\phi)}, ..., e^{j(N_{BS}-1)\frac{2\pi}{\lambda}r\sin(\phi)}]^T,$$

where λ is the signal wavelength and r is the distance between antenna elements.

At the uth user, the RF combiner $\mathbf{w}_u \in \mathbb{C}^{N_{MS} \times 1}$ is used to process the received signal , so we have

$$y_u = \mathbf{w}_u^*\mathbf{H}_u \sum_{n=1}^{U} \mathbf{F}_{RF}\mathbf{f}_n^{BB}s_n + \mathbf{w}_n^*\mathbf{n}_u,$$

where n is the complex Gaussian noise with a variance N_o.

The sum rate is given by

$$R_{sum} = \sum_{u=1}^{U} \log_2(1 + \frac{\frac{P}{U}|\mathbf{w}_u^*\mathbf{H}_u\mathbf{F}_{RF}\mathbf{f}_u^{BB}|^2}{\frac{P}{U}\sum_{n \neq u}|\mathbf{w}_u^*\mathbf{H}_u\mathbf{F}_{RF}\mathbf{f}_n^{BB}|^2) + N_o}). \qquad (2)$$

To design digital BF $\mathbf{F}_{BB}$ and analog BF $\mathbf{F}_{RF}$ $\mathbf{W}_i$ to maximize the sum rate is an optimization problem, which is given by

$$\left\{\mathbf{F}_{RF}, \{\mathbf{f}_u^{BB}\}_{u=1}^U, \{\mathbf{w}_u\}_{u=1}^U\right\} = \arg\max R_{sum}. \qquad (3)$$

Based on IEEE 802.11.ad, both $\mathbf{F}_{RF}$ and $\mathbf{w}_u$ are implemented with beamsteering codebooks [14], but $\mathbf{F}_{BB}$ is the fully digital BF. Thus, this problem is not a convex problem so that the closed form of the optimal solution is difficult to obtain.

B. Hybrid BF Algorithm

To solve the problem, a low complexity hybrid BF algorithm is proposed, which is made up of an analog BF stage and a digital BF stage.

1) Analog BF stage: Each user and its corresponding RF chain at AP select their analog BF $\mathbf{f}_u^{RF}$ and $\mathbf{w}_u$ to maximize channel gain, i.e.,

$$\arg\max ||\mathbf{w}_u^*\mathbf{H}_u\mathbf{f}_u^{RF}||.$$

In this stage, each user and its corresponding RF chain operate the BF protocol in IEEE 802.11.ad.

2) Digital BF stage: At the beginning of second stage, each user and its corresponding RF chain select the optimal analog BF to communicate. Then, each user estimates its effective channel $\bar{\mathbf{h}}_i = \mathbf{w}_i^*\mathbf{H}_i\mathbf{F}_{RF}$, then feeds it back to AP. Based on the effective CSI, AP can calculate the digital BF. Based on the zero-forcing precoding design, the digital BF is given as

$$\mathbf{F}_{BB} = \bar{\mathbf{H}}^*(\bar{\mathbf{H}}\bar{\mathbf{H}}^*)^{-1}, \qquad (4)$$

where

$$\bar{\mathbf{H}} = [\bar{\mathbf{h}}_1^T, \bar{\mathbf{h}}_2^T, ..., \bar{\mathbf{h}}_U^T]^T.$$

In this stage, each user feeds back effective CSI to AP, then AP calculates digital BF based on effective CSI.

IV. HYBRID BF PROTOCOL

In this section, we present the proposed hybrid BF protocol that is compatible with the IEEE 802.11.ad. Proposed protocol consists of multiuser SLS, multiuser ATI and multiuser BRP. As discussed in hybrid BF algorithm section, the hybrid BF contains an analog stage and a digital stage. First, multiuser SLS and multiuser BRP are performed for STAs and their corresponding RF chains to obtain the analog BF. Then, the effective CSI is estimated and fed back to AP during multiuser BRP to acquire digital BF.

A. Multiuser SLS

As shown in Fig. 5, SLS for each station is performed in each time slot to avoid interference. In IEEE 802.11.ad, the maximum number of time slots is 8.
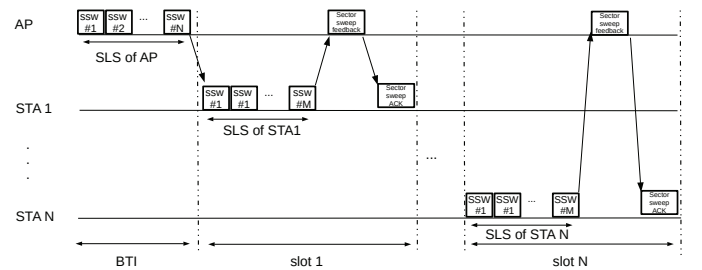

Fig. 5. Proposed multiuser SLS.

B. Multiuser ATI

ATI is used to announce channel access allocation. As shown in Fig.6, N STAs in the coverage area of AP are polled by AP. Then, each STA responds AP with a service period request (SPR) frame which contains information whether the STA requires to transmit data. Finally, AP grants U users to transmission. i.e., N STAs are polled by AP, and only U STAs are selected to transmit data simultaneously.
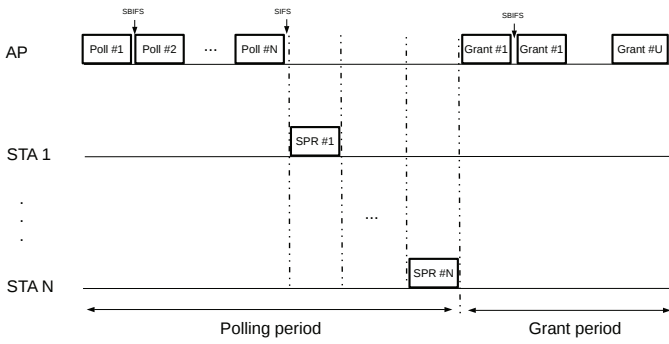
Fig. 6. Multiuser ATI.

C. Multiuser BRP

In multiuser BRP allocation, each user performs BRP with AP to obtain refined analog BF. As shown in Fig. 7, to avoid interference, BRP for different users are performed at different time slots. Besides, when each user communicates with AP with BRP frames, each user estimate the channel at the same time. Then, Each user feeds back effective CSI to AP at the end of multiuser BRP. Based on effective CSI, digital BF in (4) is acquired.
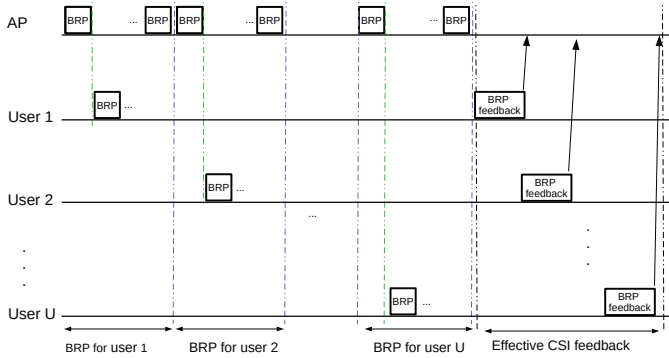

Fig. 7. Multiuser BRP.

V. OVERHEAD ANALYSIS

In a beacon interval, BF is the overhead of data transmission. In this section, we analyze the overhead of BF protocol in IEEE 802.11.ad standard and our proposed hybrid BF protocol.

A. Overhead of BF Protocol in IEEE 802.11.ad

In IEEE 802.11.ad, the overhead of BF training consists of SLS, ATI and BRP allocations. All the frames that used in BF are control frames which should be sent with the control rate. Control rate is 27.5 Mbit/s in IEEE 802.11.ad.

1) SLS Allocation: SLS consists of TXSS, RXSS, SSW feedback and SSW ACK as shown in Fig. 2. Let N_t and N_r denote the number of sectors of AP and each user, respectively. TXSS and RXSS contain N_t and N_r SSW frames, respectively. T_{SSW}, $T_{SSW-feedback}$ and $T_{SSW-ACK}$ denote the time allocation for SSW frame, SSW feedback frame and SSW ACK frame, respectively. With simple addition, the total time for SLS allocation is given by

$$T_{SLS} = N_t T_{SSW} + N_r T_{SSW} + T_{SSW-feedback} + T_{SSW-ACK}.$$

2) ATI Allocation: ATI allocation consists of polling, SPR and Grant, as shown in Fig. 6. In IEEE 802.11.ad, only one user is served at each time slot. Thus, we only have one polling frame, one SPR frame and one Grant frame. $T_{polling}$, T_{SPR} and T_{Grant} denote the time used for a polling frame, a SPR frame and a Grant frame , respectively. The total time of ATI allocation is given by

$$T_{ATI} = T_{polling} + T_{SPR} + T_{Grant}.$$

3) BRP Allocation: As the BRP example shown in Fig. 3, BRP allocation consists of BRP setup, MID, BC and BRP transaction. MID and BC are iterative BRP training. There are three kinds of BRP frames: BRP frame, BRP feedback frame and BRP with TRN-T/R frame. BRP feedback frame is the BRP frame appended with channel measurement information. BRP with TRN-R/T frame is the BRP frame appended with TRN-R/T field to perform channel measurement. In Fig. 3, components of each subphase are as follows,

- BRP setup: 5 BRP frames;
- MID: 2 BRP with TRN-R/T frames and 2 BRP feedback frames;
- BC: $2N_{beam}$ BRP with TRN-R/T frames and 2 BRP feedback frames;
- BRP transaction: One BRP frame and 5 BRP with TRN-R/T field frames.

Total time of BRP is

$$T_{BRP} = T_{BRP-setup} + N_{BRP}(T_{MID} + T_{BC}) + T_{BRP-tran},$$

where N_{BRP} represents the number of BRP iterations.

The total overhead of BF in IEEE 802.11.ad is the summation of SLS, ATI and BRP, which is given by

$$T_{overhead}^s = T_{SLS} + T_{ATI} + T_{BRP}. \tag{5}$$

B. Overhead of Proposed Hybrid BF Protocol

As discussed in hybrid BF protocol section, hybrid BF overhead consists of multiuser SLS, multiuser ATI and multiuser BRP.

1) Multiuser SLS Allocation: As shown in Fig. 5, we have multiple RXSS, SSW feedback and SSW ACK to support N stations in the coverage area of AP. The total time of multiple SLS is

$$T_{SLS}^m = N_t T_{SSW} + N(N_r T_{SSW} + T_{SSW-feed} + T_{SSW-ACK}).$$

2) Multiuser ATI Allocation: As shown in Fig. 6, consider N STAs in the coverage area of AP are active, N STAs are polled by AP and only U users are granted with the service period (SP). The selected U users transmit data simultaneously in the SP. The total time of ATI is

$$T_{ATI}^m = N(T_{polling} + T_{SPR}) + T_{Grant}U.$$

3) Multiuser BRP Allocation: For multiuser BRP allocation, BRP is performed for each user at each time slot, so the total time is

$$T_{BRP}^m = T_{BRP}U.$$

Above equation illustrates that time consumption of multiuser BRP increases linearly with the number of users. As BRP

occupies most of the BF time, it implies that overhead of hybrid BF protocol increases nearly linearly with the number of users.

The total overhead of BF in our proposed strategy is

$$T_{overhead}^m = T_{SLS}^m + T_{ATI}^m + T_{BRP}^m. \quad (6)$$

VI. SIMULATIONS RESULTS

We consider an indoor scenario, AP has a coverage area within radius of R. N STAs are uniformly distributed in the coverage area of AP. We randomly choose U users to perform hybrid BF so that U users can transmit data simultaneously.

For the path loss, we consider STA-AP LOS link channel model. The average path loss can be represented by

$$Path - loss(dB) = A + 20\log_{10}(f) + 10\gamma\log_{10}(d), \quad (7)$$

where $A = 32.5$, f is the frequency in GHz and $\gamma = 2$ [15].

The simulation parameters are given in Table. I. The beacon interval length is 1 second in our simulations. AP has 16 transmitting sectors and each user has 8 receiving sectors. The radius of the coverage area of AP is 5 meters.

In Fig. 8, we compare the overhead of BF with different number of users. The overhead for BF in IEEE 802.11.ad which supports single user transmission is given in (5) and the overhead of hybrid BF in our proposed strategy is given in (6). Overhead increases linearly with the number of users because multiuser BRP is implemented at different time slots for each user. For example, for a 8-user hybrid BF case, the overhead of BF is 400 ms which occupies about 40 % of the whole beacon interval. Results suggest hybrid BF is inefficient in supporting a large number of users in IEEE 802.11.ad.

TABLE I
SIMULATION PARAMETERS.

| Parameter	value
SSW frame	26 bytes
SSW feedback frame	28 bytes
SSW ACK frame	28 bytes
Polling frame	22 bytes
SPR frame	27 bytes
Grant frame	27 bytes
BRP frame	986 bytes
BRP feedback frame	1008 bytes
BRP with TRN-R/T frame	2830 bytes
control rate	27.5 Mbit/s
Beacon interval	1000 ms
Background noise	-134 dBm/MHz
bandwidth	2.16 GHz
central frequency	60 GHz
N_{BS}	16
N_{MS}	8
N_t	16
N_r	8
N_{BRP}	4
N_{beam}	4
N	8
U	4
R	5 meters
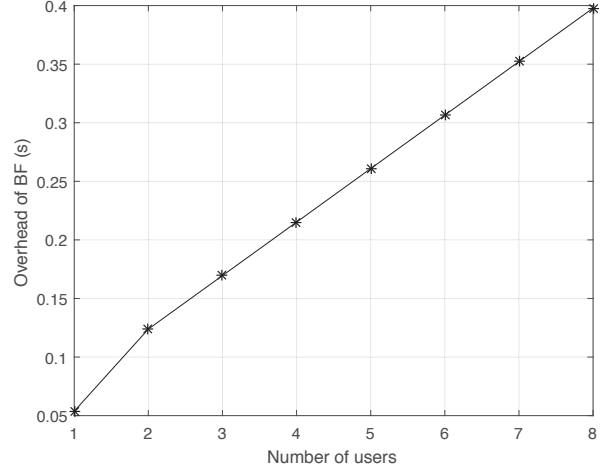
Fig. 8. Time cost of BF training with different number of users.

Here we define the effective rate as

$$R_e = \frac{T - T_{overhead}}{T}R_{sum}, \quad (8)$$

where R_e is the effective rate, $T_{overhead}$ is the time of BF overhead and R_{sum} is the raw transmission rate in (2).

Considering the overhead of BF, we compare the effective rates of hybrid BF with respect to different transmitting power, as shown in Fig. 9. Simulations demonstrate that hybrid BF can achieve a significant throughput gain compared with the single user transmission in IEEE 802.11.ad because hybrid BF can exploit multiuser multiplexing gain. However, as more users perform hybrid BF, the overhead of BF increases significantly. It can be seen from Fig. 8 that given the transmission power at 0.001mW, 5-user hybrid BF outperforms others. Thus, It's very important to find the optimal number of users to perform hybrid BF with different transmitting power.
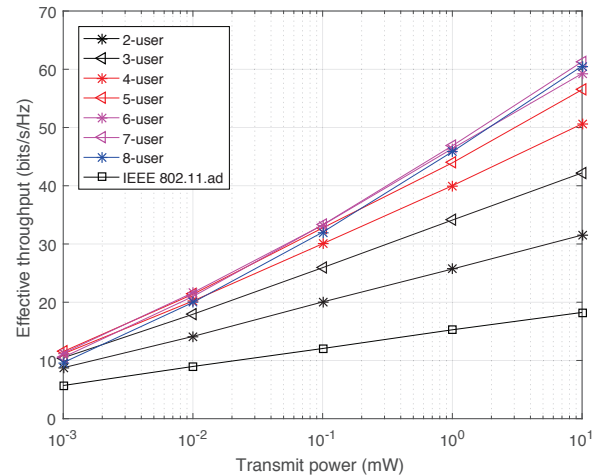

Fig. 9. Effective sum rate comparison of hybrid BF with different number of users.
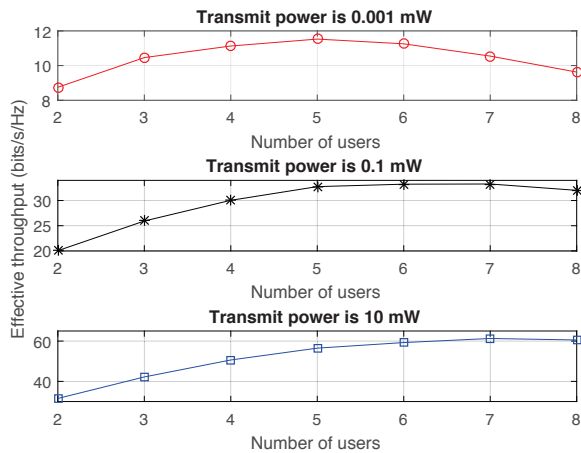
Fig. 10. Effective rate comparison of hybrid BF with different number of users with given transmitting power.

In Fig. 10, we compare the effective rates of hybrid BF with different number of users given transmitting power is 0.0001mW, 0.1mW and 10mW. When transmitting power is low, e.g., 0.001mW, the optimal number of users is 5. When transmitting power is 0.1 mW, the optimal number of users is 6. With high transmitting power, e.g., 10mW, the optimal number of users is 7. The optimal number of users increases with strength of transmitting power.

VII. Conclusion

In this paper, we have proposed a hybrid BF protocol based on IEEE 802.11.ad to implement downlink multiuser transmission. Then, we have analyzed the overhead of our proposed hybrid BF protocol and found that overhead increases nearly linearly with the number of users. Considering overhead of BF, hybrid BF strategy could still achieve a significant throughput gain compared with that in IEEE 802.11.ad. Theoretical results show that the throughput gain achieved by increase of users could be degraded by increasing overhead. Our work contributes to the system design of optimal downlink multiuser transmission in mmWave WiFi.

For future work, we plan to investigate the performance of BF protocol in IEEE 802.11.ad in a high mobility scenario.

VIII. Acknowledgment

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